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The role of competitiveness in the Prisoner's Dilemma

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Abstract

Background: Competitiveness is a relevant social behavior and in several contexts, from economy to sport activities, has a fundamental role. We analyze this social behavior in the domain of evolutionary game theory, using as reference the Prisoner's Dilemma.

Methods: In particular, we investigate whether, in an agent population, it is possible to identify a relation between competitiveness and cooperation. The agent population is embedded both in continuous and in discrete spaces, hence agents play the Prisoner's Dilemma with their neighbors. In continuous spaces, each agent computes its neighbors by an Euclidean distance-based rule, whereas in discrete spaces agents have as neighbors those directly connected with them. We map competitiveness to the amount of opponents each agent wants to face; therefore, this value is used to define the set of neighbors. Notably, in continuous spaces, competitive agents have a high interaction radius used to compute their neighbors. Instead, since discrete spaces are implemented as directed networks, competitiveness corresponds to the out-degree of each agent, i.e., to the number of arrows starting from the considered agent and directed to those agents it wants to face.

Results and conclusions: Then, we study the evolution of the system with the aim to investigate if, and under which conditions, cooperation among agents emerges. As result, numerical simulations of the proposed model show that competitiveness strongly increases cooperation. Furthermore, we found other relevant phenomena as the emergence of hubs in directed networks.

Keywords: Cooperation, Competitiveness, Social dynamics, Prisoner's Dilemma

Background

In the last years, social and economic phenomena have attracted the interest of scientists belonging to hard sciences, as mathematics, physics and computer science. As result, the interdisciplinary fields of social dynamics [1, 2] and econophysics [3] have rapidly emerged. For instance, several analytical and computational approaches have been developed for studying behaviors such as homophily [4], conformity [5–8], and rationality [9, 10]. Furthermore, many social and economic phenomena can be studied in the context of Evolutionary Game Theory [11–13], which represents the attempt of describing the evolution of populations by Game Theory using famous models like the Prisoner's Dilemma [14, 15] (PD hereinafter). Since the PD allows to analyze the phenomenon of cooperation [16–18], it is possible to study the evolutionary dynamics among agents whose interactions are based on this game. In doing so, we can evaluate if, and under which conditions, cooperation emerges. It is worth to highlight that simple games like the PD, implemented considering different social behaviors, contexts (see [19]), or topologies (e.g., [20–24]) to implement agent's interactions, as sketched before, allow to investigate a wide variety of topics such as criminality [25], biological systems [26], imitation phenomena [27], and further social psychology aspects such as conformity [28, 29]. Here, we consider an important social character, i.e., the competitiveness, that strongly affects dynamics in animal herds and among individuals [4]. In particular, in this study, we aim to investigate if there is a relation between competitiveness and cooperation. To this end, we implement a population whose agents, provided with a parameter that represents their degree of competitiveness (see [30]), play the PD. The relevance of this work lays in the fact that, both in herds and in human communities, many contexts are defined as competitive, e.g., stock markets, athletic challenges, and job markets. Numerical simulations, of the proposed model, allowed to analyze parameters as the average out-degree over time and to define the TS-diagram; the latter constitutes a relevant tool to assess if, and in which extent, cooperation emerges among agents. As result, we found that competitiveness strongly affects these dynamics and, in particular, it increases the cooperation among agents. The remainder of the paper is organized as follows: "Model" introduces the model for studying the PD in continuous spaces and in discrete spaces. "Results" shows results of numerical simulations on varying the initial conditions. Eventually, "Discussion and conclusion" ends the paper.

Model

In the proposed model [30], we study a population, embedded in a bidimensional continuous space and in a discrete space, whose agents play the PD. The continuous space is represented by a square of side $L = 1$, where agents are equally spread inside it. Instead, the discrete space is represented by a directed network of agents. In so doing, agents play the PD with their neighbors: (a) in the continuous space, neighbors are computed by an Euclidean distance-based rule [31], whereas (b) in the discrete space, each agent has as neighbors those connected by an arrow (starting from the considered agent). It is worth to emphasize that, since we are dealing we a directed network, for each pair of agents—say A and B, there is a reciprocal interaction only if there are two arrows: one from A to B, and one from B to A. Therefore, if there is only one arrow between A and B, e.g., from A to B, the B agent is a neighbor of A, but A is not considered a neighbor of B. These relations appear clear considering that the related adjacency matrix, i.e., the matrix containing all the information about the connections, is not symmetric as for undirected networks (e.g., friendship networks and collaborator networks [7]). In principle, the PD is a very simple game where agents may behave as cooperators or as defectors, then in accordance with a payoff matrix, they increase or decrease their payoff when they face each other. In particular, depending on their behavior and on that of their opponents, agents compute their gain at each interaction. Moreover, it is worth to note that, in this context, to behave as a cooperator means to adopt a cooperation strategy and, in the same way, to behave as a defector means to play with a defection strategy. The way agents update their payoff, in accordance with their behavior (i.e., strategy), is described in the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & s \\ T & 0 \end{pmatrix} \end{array} \tag{1}$$

The set of strategies is $\Sigma = \{C, D\}$, where C stands for 'Cooperator' and D for 'Defector'. In the matrix 1, T represents the *Temptation*, i.e., the payoff that an agent gains if it defects while its opponent cooperates, while S the *Sucker's payoff*, i.e., the gain achieved by a cooperator while the opponent defects. In the PD, game values of T and S are in the following range: $1 \leq T \leq 2$ and $-1 \leq S \leq 0$. As discussed before, the TS-plane is a relevant tool while studying the system because, as we can see in matrix 1, the PD can be played with different values of S and T, having different meanings. For instance, a low value of T entails defectors have a small increase of their payoff when they play against cooperators, whereas a high value of S entails small losses for cooperators which play against defectors. Therefore, it is interesting to investigate whether a cooperative behavior emerges, in the agent population, on varying the values of described parameters (i.e., T and S). In general, the evolution of a population can be simulated in two different ways: synchronous dynamics or asynchronous dynamics. The former entails that at each time step, all agents interact (i.e., they play the PD with their neighbors). Instead, the latter entails that at each time step only one agent is considered, i.e., it computes its neighbors and faces them playing the PD. Remarkably, in this work, simulations have been implemented by the asynchronous dynamics. To summarize, the main steps of the proposed model are:

1. A randomly chosen agent, say the jth agent, computes the set of its neighbors in accordance with the interaction radius r (or with the network structure in the discrete space);
2. The jth agent faces its neighbors (note that each single challenge involves only two agents at time);
3. All agents, playing at this step (i.e., the jth agents and its neighbors), compute their new payoff;
4. The jth agent updates its strategy according to a revision rule.

In doing so, each agent involved in the game obtains a payoff in accordance with its strategy (i.e., cooperation or defection), considering the payoff matrix 1. Now, let $\sigma_j(t)$ be a vector giving the strategy profile of the jth agent at time t with $C = (1, 0)$ and $D = (0, 1)$, and let M be the payoff matrix discussed above. The payoff collected by the jth agent, at time t, can be computed as

$$\Pi_j(t) = \sum_{i \in N_j} \sigma_j(t) M \sigma_i^{\top}(t). \quad (2)$$

In the proposed model, we adopted the strategy revision rule called 'imitation of the best': the jth agent compares its payoff (P_j) with those of its neighbors, and it adopts the strategy of the neighbor having the highest payoff if it is greater than P_j. As a consequence, agents can vary their strategy several times during the evolution of the system. Since some parameters of the proposed model depend on the considered domain (i.e., continuous and discrete), we illustrate both cases with more detail.

Continuous space

As shown in [32], using low values of T and high values of S, cooperation among agents emerges only under particular conditions, i.e., when agents randomly move over time.

It is worth to highlight that in [32], all agents have the same radius to compute the set of their neighbors. Furthermore, this radius depends on the average number of opponents agents face. Here, we consider the same geometrical framework (i.e., that defined in [32]) to implement the proposed model on continuous spaces, with two main differences: (1) agents are fixed (i.e., they cannot move) and (2) agents can vary their radius. Notably, agents have an interaction radius whose length depends on gained payoff: as their payoff increases/decreases their radius increases/decreases. Hence, agents with high payoff become more competitive and, as result, they face a higher number of opponents than agents with a small payoff. At time $t = 0$, all agents have the same radius computed according to the average number of opponents they can face (if selected). In particular, the radius $r(t = 0)$ is computed as $r(0) = \sqrt{\bar{k(0)}/(\pi N)}$. Then, considering that each radius varies in accordance with agent's payoff, and that agents face a number of opponents in the range $[1, N-1]$, the radius is computed as $r = \alpha r_0$, where $(\sqrt{1/\bar{k}}) \leq \alpha \leq \sqrt{(}N/\bar{k})$. Thus, at $t = 0$, the value of α is $\alpha_0 = 1$. In general, after n time steps, each agent plays an average number of times equal to $\bar{n} = n/N$. Since best agents (i.e., those with high payoff) should get the maximum radius in $\bar{n}$ steps, every time agents play, their value of α increases to $\delta\alpha = (\alpha_{\max} - \alpha_0)/\bar{n}$. Hence, the radius is modified to $\pm\delta r$, where $\delta r = r_0 \delta\alpha$, depending on which the considered agent obtains a positive or a negative payoff.

Discrete space

The discrete space is implemented by a directed network, i.e., a network whose connections can be represented by arrows. In the proposed model, an arrow from one agent to another one represents the challenger agent (i.e., the one that faces someone else) and the faced agent (i.e., agent identified as neighbor of the challenger one). In directed networks, the definition of neighbors is not immediate as for undirected networks, where connections can be represented by simple lines. Notably, arrows represent links (or edges) and their direction represents the meaning of the relation. For instance, an arrow starting from node A, and ending to node B, codifies a relation from A to B, and not vice versa. Thus, neighbors of the jth node are those nodes connected to it by arrows starting from the jth node itself. In doing so, an arrow starts from the challenger and it ends on the faced agent. To analyze the structure of these networks, using the degree distribution, we have to consider both the "in-degree" distribution and the "out-degree" distribution. The former represents the distribution of links ending in nodes, whereas the latter those of links starting from nodes. Then, competitiveness can be mapped to the out-degree of each node. As for the continuous space, at $t = 0$ all agents begin to play in the same conditions, i.e., all nodes have the same out-degree and the same in-degree. On the other hand, as the population evolves (i.e., agents play the PD over time), winning agents increase their out-degree (randomly selecting new opponents) and loosing agents do the opposite, i.e., they reduce their out-degree (randomly selecting nodes to remove from their neighborhood). As before, the increment/reduction of the out-degree has as constraint that each agent cannot play with more than $N - 1$ agents nor less then 1 agent. Furthermore, the increasing and the decreasing is unitary, i.e., the k_{out} can vary at each time step of ± 1. Finally, we recall that in both domains we adopted the 'imitation of the best' strategy revision rule, and in all simulations we consider an equal initial distribution of strategies, i.e., at the beginning the 50% of the population is composed of cooperators and the remaining 50% of defectors.

Results

We performed many numerical simulations to study the evolution of the system and, moreover, we highlight that each presented result has been obtained by averaging over 50 different simulation runs. In particular, we investigated the following cases:

- Mean-field approximation
- Continuous spaces
- Discrete spaces

The first case represents a classical generalization of the studied system, as we introduce the trivial hypothesis that all agents interact with all the others, at each time step. In terms of networks theory, this scenario corresponds to a fully connected network, hence complex interaction patterns are not considered nor the competitiveness is represented. Notably, competitiveness is mapped to the number of opponents each agent faces; therefore, in the event everyone faces everyone, competitiveness vanishes. Anyway, when studying complex systems, before focusing on complex scenarios it is often useful to analyze results coming from simple or trivial configurations. Then, once we performed the first analysis, we proceed on analyzing results related to the continuous space and to the discrete space.

Mean-field approximation

Here, we consider a simple fully connected network structure to arrange agents. We observe that this kind of configuration can be studied also in a continuous domain, making the assumption that every agent is provided with an interaction radius long enough to include in its social circle all the other agents. Both implementations, of the mean-field approximation, are equivalent as both produce the same effects on agents. As shown in Figure 1, in the event agents interact with all the population, at the same time and without considering particular characters as the competitiveness, the population reaches always the same final defection phase, i.e., all agents behave as defectors for every value of T and S. Only for very high values of S and for low values of T, a small amount of cooperative agents survives.

Anyway, it is possible that, if we observe the evolution for a time longer than 10^4 time steps, all agents of the population become defectors. In general, this first result confirms that in absence of particular behaviors (e.g., movements and social characters) the defection strategy dominates, according to the expected Nash equilibrium. Hence, we can go ahead studying the population by introducing the competitiveness.

Simulations on the continuous space

We recall that the continuous space is represented by a bidimensional square of side $L = 1$. In this geometrical configuration, we spread $N = 100$ agents by two different ways: uniform distribution and regular lattice distribution. The former entails the distribution is completely random in the space, whereas the latter entails agents can occupy

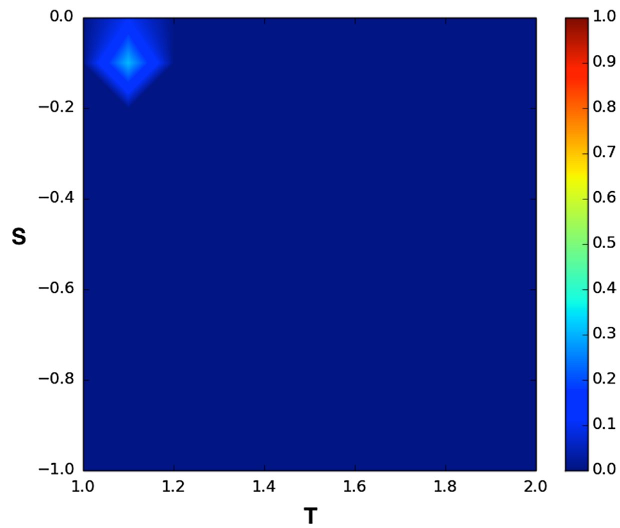

Figure 1 Mean-field approximation. Cooperation frequencies in the *TS*-plane achieved by a population arranged on a fully connected network. This result is in full accordance with the expected Nash equilibrium for the PD. Parameter *S* indicates the payoff obtained by cooperators that face defectors, that in turn gain a payoff equal to *T* (when facing cooperators)—see matrix 1. *Colors* indicate the averaged degree of cooperation achieved by the population. We recall that *red* indicates strong cooperation, while *blue* defection (i.e., no cooperation).

specific positions, forming a bidimensional lattice. We consider two different conditions related to the initial average degree: $\bar{k}(0) = 4$ and $\bar{k}(0) = 8$. Then, we provide agents with a radius $r_0 = \sqrt{\bar{k}(0)/(\pi N)}$. In doing so, at the beginning all agents have the same radius. Results related to the uniform distribution are shown in Figure 2, while those related to the population arranged on a regular lattice (embedded in the continuous space) are shown in Figure 3.

Observations of these diagrams in Figures 2 and 3 let emerge that when agents have a higher initial average degree the final density of cooperators decreases. Furthermore, it

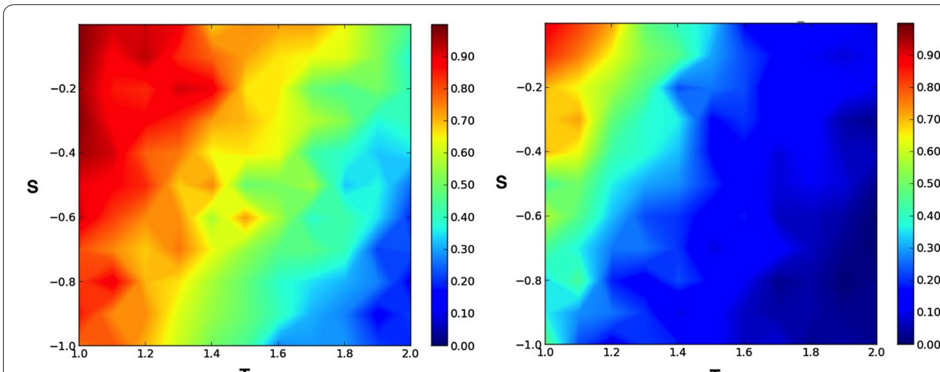

Figure 2 Continuous space: uniform distribution. Cooperation frequencies in the *TS*-plane. On the *left*, results achieved using agents provided with $\bar{k}(0) = 4$. On the *right*, results achieved using agents provided with $\bar{k}(0) = 8$. Parameter *S* indicates the payoff obtained by cooperators that face defectors, that in turn gain a payoff equal to *T* (when facing cooperators)—see matrix 1. *Colors* indicate the averaged degree of cooperation achieved by the population. We recall that *red* indicates strong cooperation, while *blue* defection (i.e., no cooperation).

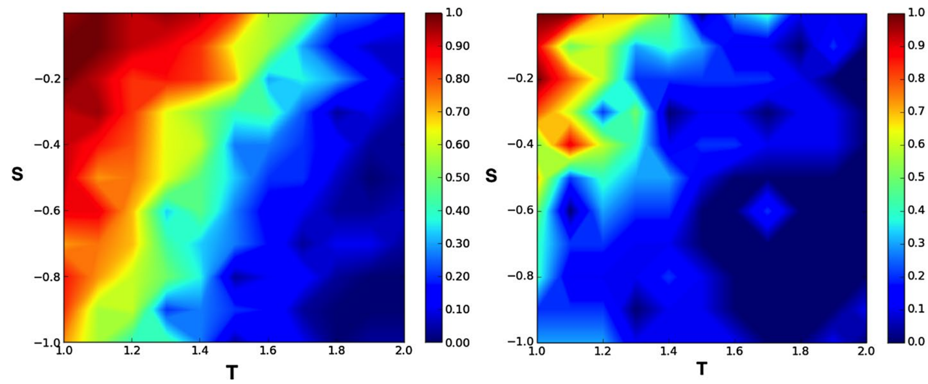

Figure 3 Continuous space: lattice distribution. Cooperation frequencies in the *TS*-plane. On the *left*, results achieved using agents provided with $\bar{k}(0) = 4$. On the *right*, results achieved using agents provided with $\bar{k}(0) = 8$. In both cases, $\bar{k}(0)$ refers to $\bar{k}(0)_{in}$ and $\bar{k}(0)_{out}$. Parameter *S* indicates the payoff obtained by cooperators that face defectors that in turn gain a payoff equal to *T* (when facing cooperators)—see matrix 1. *Colors* indicate the averaged degree of cooperation achieved by the population. We recall that *red* indicates strong cooperation, while *blue* defection (i.e., no cooperation).

is relevant to emphasize that by arranging agents in a regular lattice, with 4 and 8 neighbors, when they increase/decrease their radius the variation of faced opponents is equal to their initial average degree, i.e., ± 4 and ± 8, respectively.

Simulations on the discrete space

We recall that the discrete space is represented by a directed network. Notably, we implemented this scenario using a regular lattice as initial configuration. In this case, we were able to consider a population with $N = 1,000$ agents, comparing the case with agents having a fixed out-degree and variable out-degree. The former constitutes a scenario equivalent to that given by agents with fixed radius in the continuous domain, whereas the latter corresponds to a variable radius (in the continuous domain). Furthermore, due to the increasing of k_{out} over time for competitive agents (and to the decreasing of the same parameter for non-competitive agents), we are dealing with adaptive networks (see [33]), i.e., networks whose structure varies over time. Results of simulations are shown in Figure 4.

Then, we analyzed the degree distributions (both the in-degree and the out-degree distribution) of resulting networks, choosing representative points of the *TS*-plane. Figure 5 shows the degree distributions for a cooperation region (of the *TS*-plane), and Figure 6 shows degree distributions achieved in a defection region.

It is worth to see how the in-degree distributions vary much lesser than the out-degree distributions, although both are involved in the evolution of the system.

Discussion and conclusion

In this study, we aim to investigate if there are relations between two social behaviors, i.e., cooperation and competitiveness, when an agent population evolves playing the Prisoner's Dilemma. In particular, we map the competitiveness to a parameter embedded in the model, so that competitive agents face many opponents, whereas non-competitive ones do the opposite. In the proposed model, becoming a non-competitive agent

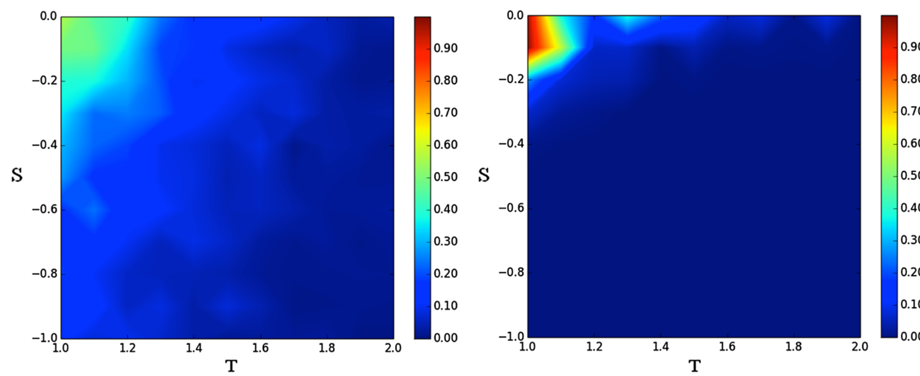

Figure 4 Discrete space. Cooperation frequencies in the *TS*-plane. On the *left*, results achieved using agents provided with constant out-degree, i.e., a scenario equivalent to 'constant radius' in the continuous domain. On the *right*, results achieved using agents provided with a variable out-degree (i.e., equivalent to variable radius)—see [30]. Parameter *S* indicates the payoff obtained by cooperators that face defectors that in turn gain a payoff equal to *T* (when facing cooperators)—see matrix 1. *Colors* indicate the averaged degree of cooperation achieved by the population. We recall that *red* indicates strong cooperation, while *blue* defection (i.e., no cooperation).

entails to loose challenges, while playing the Prisoner's Dilemma. After performing a brief mean-field analysis of our model, where the population reached the expected Nash equilibrium, agents have been arranged in two different domains: a continuous space and a discrete space. The former is represented by a bidimensional square, whereas the latter has been modeled by a directed network. First of all, we highlight the main differences between our work and those performed by previous authors (e.g., [31, 32, 34]): we focus our attention on fixed agents and we provide them with a social character, i.e., the competitiveness. Due to the computational cost of our model, we were able to perform simulations up to $t = 10^4$ time steps, with $N = 100$ agents in the continuous space and with $N = 1,000$ agents in the discrete space. In general, the main result of numerical simulations shows that competitiveness allows the emergence of cooperation areas in the *TS*-plane, in both domains. Moreover, in the continuous domain, we investigated the outcomes on varying the initial conditions: the spreading of agents in the bidimensional square (i.e, random vs regular lattice) and the average degree (i.e., $\bar{k}(0) = 4$ and $\bar{k}(0) = 8$). Notably, when agents are randomly spread, several intermediate phases are obtained, indicating an equal presence of cooperators and defectors, instead by an ordered distribution (i.e., lattice) we found more neat areas of cooperation and defection. On the other hand, the initial average degree seems to have a strong influence on these dynamics, as for $\bar{k}(0) = 4$ the cooperation area in the *TS*-plane is greater than for $\bar{k}(0) = 8$, using the two spreading strategies. This difference can be explained by the fact that, as for each agent the number of neighbors increases (at $t = 0$), the probability that the related social circle be composed of cooperators (i.e., be a cluster of cooperative agents) reduces. In the discrete domain, the scenario is a bit different as only for very low T values and high S values, a full cooperation emerges. An analysis related to the influence of the initial arrangements of agents, in both domains, performed to understand why some of them appear more advantageous to obtain more cooperation is important and it will constitute the argument for future investigations. Finally, we analyzed the degree distributions

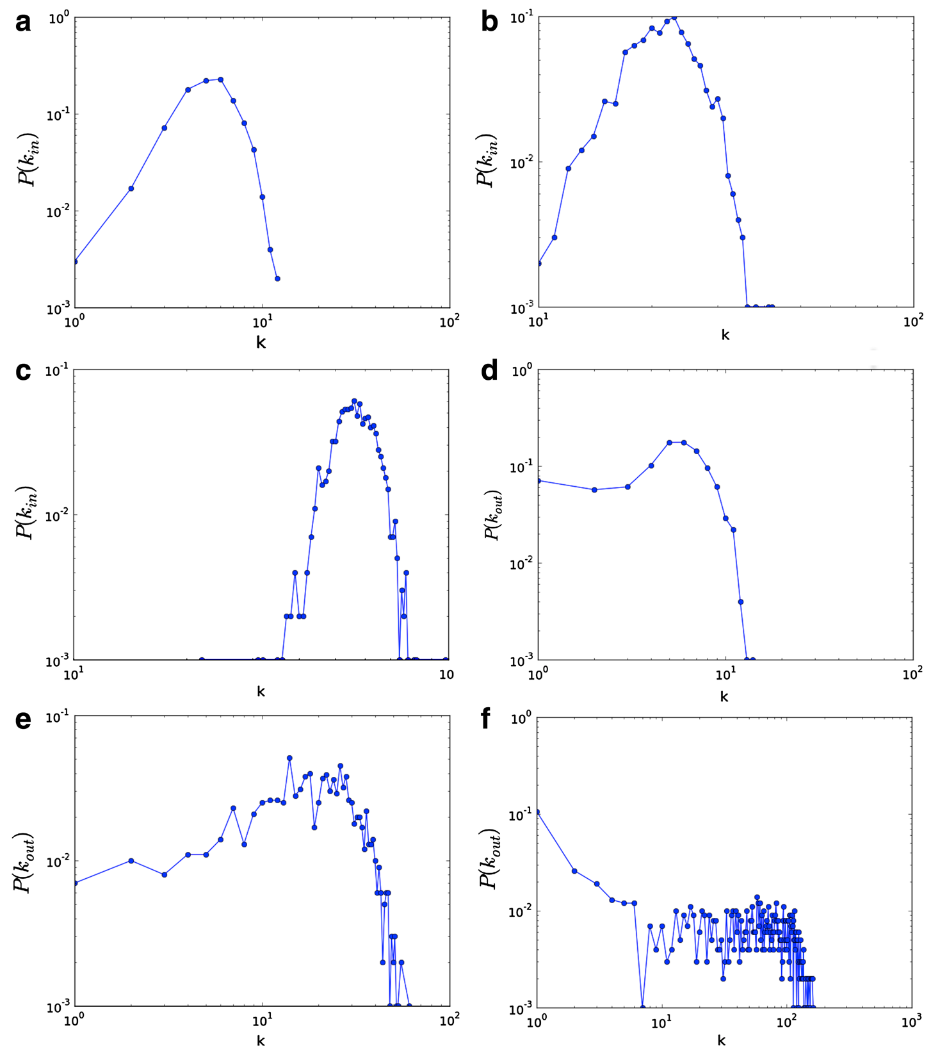

Figure 5 Degree distributions achieved in networks of cooperative agents (selected according to the *TS*-plane). In-degree distributions $P(k_{\text{in}})$: **a** at $t = 1,000$. **b** At $t = 5,000$. **c** At $t = 10,000$. Out-degree distributions $P(k_{\text{out}})$: **d** at $t = 1,000$. **e** At $t = 5,000$. **f** At $t = 10,000$.

(i.e., the in-degree and the out-degree distributions) of directed networks. This analysis is relevant as agents can vary their in-degree distribution and out-degree distribution as result of their behavior (more competitive or not). It is important to note that the in-degree distribution has low variations over time, whereas the opposite happens for the out-degree distribution. Notably, this latter represents the competitive parameter, i.e., the number of opponents that competitive agents face as their payoff increases. Analyzing networks related to cooperation areas, in the *TS*-plane, we found that the out-degree distribution is characterized by the presence of more hubs (i.e., many competitive agents appear, even if they tend to cooperate among themselves). On the other hand, considering networks-related non-cooperative areas, of the *TS*-plane, we found only few variations of the out-degree distribution. In our view, this difference between the two areas, considering the out-degree distributions, means that when agents cooperate the network loses its homogeneous structure (recall that at $t = 0$ all agents have the same values of k_{in}

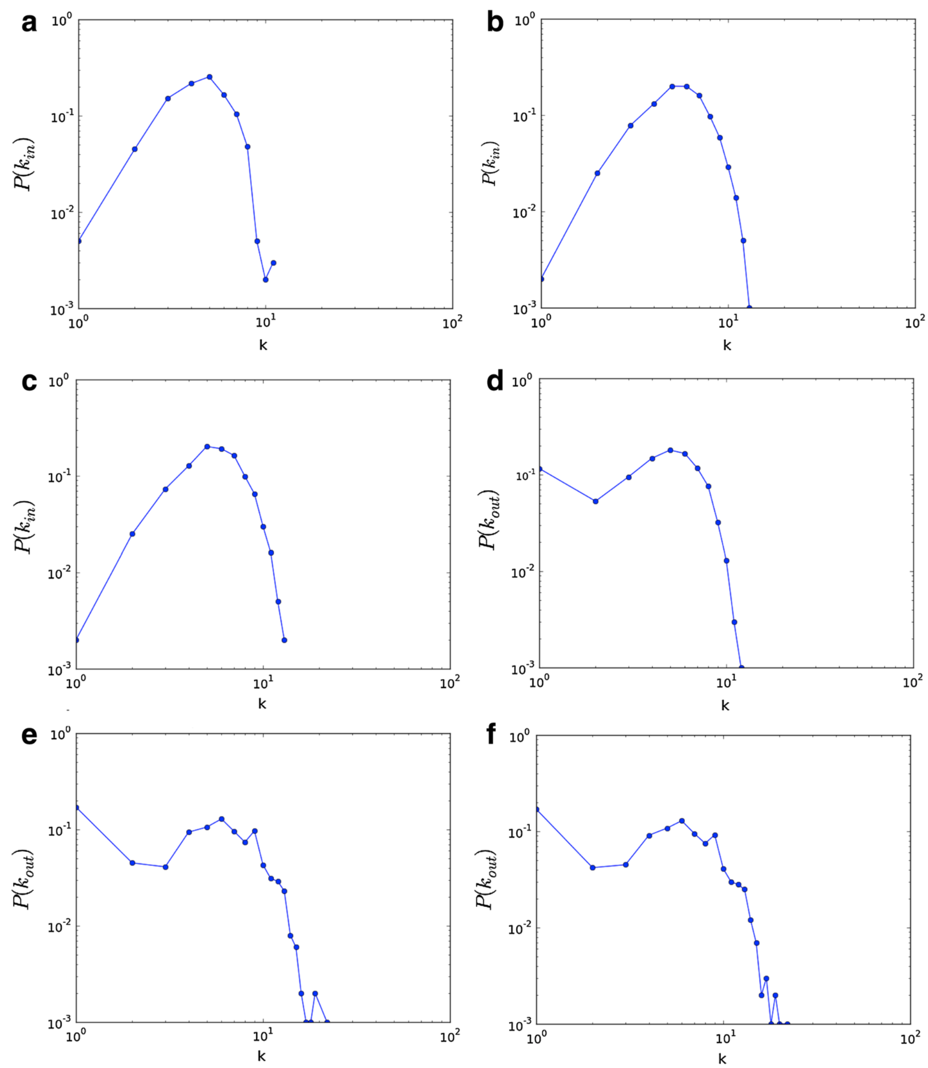

Figure 6 Degree distributions achieved in networks of non-cooperative agents (selected according to the *TS*-plane). In-degree distributions $P(k_{\text{in}})$: **a** at $t = 1,000$. **b** At $t = 5,000$. **c** At $t = 10,000$. Out-degree distributions $P(k_{\text{out}})$: **d** At $t = 1,000$. **e** At $t = 5,000$. **f** At $t = 10,000$.

and k_{out}); while when agents do not cooperate, the network structure has an exponential degree distribution (i.e., the homogeneous structure is conserved over time). In the light of these results, we can state that competitiveness strongly affects cooperation. Therefore, it is important trying to explain the underlying mechanism that leads to this result. Let us consider first the continuous case, where agents are fixed and, according to previous works, should not cooperate. Now, if only few of them have many cooperative agents in their neighborhood, they increase their interaction radius. Hence, they face more agents during next time steps, having the opportunity to face other cooperative agents. Now, according to the matrix 1, clusters of cooperators strongly increase their payoff, while clusters of defectors do not increase it in absence of cooperators. Since cooperators are randomly spread in the space, increasing the interaction radius the probability to find cooperators increases. On the other hand, defector agents, although never decrease their radius, may increase their payoff (and their radius) only for high values of T, otherwise they will have a constant small radius and, as a consequence, a small degree of competitiveness. Similar considerations hold also for the discrete domain, where defectors do not increase their out-degree, while cooperators have this opportunity. To conclude, we highlight that achieved results clearly indicate the existence of a relation between competitiveness, interpreted as an inclination to face many players, and the emergence of cooperation in the Prisoner's Dilemma.

Authors' contributions

MAJ devised the research work. Both authors performed experiments and analyzed the outcomes. Both authors read and approved the final manuscript.

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Compliance with ethical guidelines

Competing interests

The authors declare that they have no competing interests.

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